Answer ALL questions. Write your answers in the spaces provided.

1.

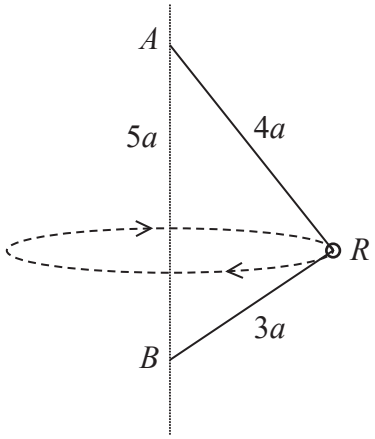

Figure 1

A small smooth ring, R, of mass m is threaded on a light inextensible string of length $7a$. The string has its ends attached to two fixed points, A and B, where A is vertically above B and $AB = 5a$. The ring moves in a horizontal circle about AB, with the string taut and $AR = 4a$, as shown in Figure 1.

The ring moves with constant angular speed ω.

Find the magnitude of ω, giving your answer in terms of a and g.

(6)

Question 1 continued

(Total for Question 1 is 6 marks)

2.

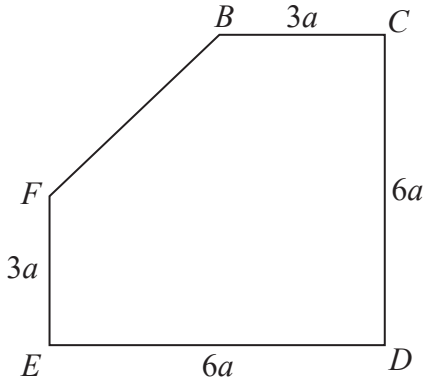

Figure 2

The uniform lamina, L, with $BC = EF = 3a$ and $CD = DE = 6a$, shown in Figure 2, is formed by removing an isosceles triangle, whose equal sides are of length $3a$, from a square of side $6a$.

(a) Show that the centre of mass of L is $\frac{23}{7}a$ from EF.

(4)

A uniform triangular lamina, ABF, made from a different material to L, is fixed to L to form the square template, $ABCDEF$, shown in Figure 3.

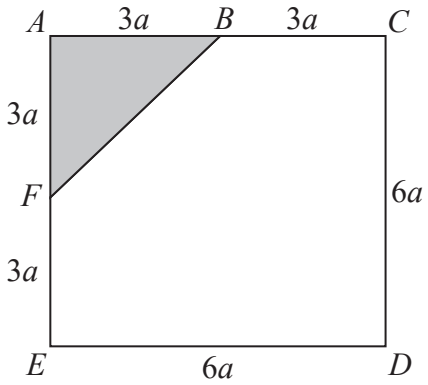

Figure 3

The lamina L has mass M per unit area. The triangular lamina ABF has mass $3M$ per unit area.

The template is freely suspended from C and hangs in equilibrium, with CD at an angle $\theta°$ to the downward vertical.

(b) Find the value of θ.

(6)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 10 marks)

3. A particle, P, moves in a straight line with simple harmonic motion about a fixed centre O. The maximum speed of P during the motion is $1.2\,\text{m}\,\text{s}^{-1}$. The maximum magnitude of the acceleration of P during the motion is $4.8\,\text{m}\,\text{s}^{-2}$.

Find

(a) (i) the period of the motion,

(ii) the amplitude of the motion.

(5)

(b) the speed of P when P is 0.1 m from O

(2)

(c) the length of time within one oscillation for which P is less than 0.1 m from O

(3)

Question 3 continued

(Total for Question 3 is 10 marks)

4. A tree trunk is 6 m long and has mass 630 kg.

The tree trunk is modelled as a non-uniform rod such that, at a distance x metres from the base of the tree trunk, the mass per unit length of the tree trunk, λ kg m^{-1}, is given by the formula $\lambda = k\left(2 - \frac{x}{12}\right)$, where k is a positive constant.

(a) Show that $k = 60$

(3)

The tree trunk rests horizontally in equilibrium on two supports, each of which is 2 m from an end, as shown in Figure 4.

Figure 4

(b) Using the model, find the magnitude of each of the reactions between the tree trunk and the supports.

(8)

Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 11 marks)

5. A particle, P, is moving along the positive x-axis. At time t seconds, $OP = x$ metres, the velocity of P is $v\,\text{m s}^{-1}$ in the direction of x increasing, and the acceleration of P is $\left(\frac{10}{x^2} - \frac{4}{x^3}\right)\text{m s}^{-2}$, where $x \geqslant 1$, in the direction of x increasing.

At time $t = 0$ seconds, $x = 1$ and $v = 3$

(a) Show that $v = \frac{5x - 2}{x}$, justifying your answer carefully.

(6)

At time T seconds, $x = 4$

(b) Show that $T = \frac{3}{5} + \frac{2}{25}\ln 6$

(6)

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Question 5 continued

(Total for Question 5 is 12 marks)

6.

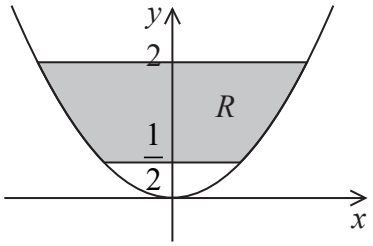

Figure 5

The region R, shown shaded in Figure 5, is bounded by the curve with equation $y = \frac{1}{2}x^2$, the line with equation $y = 2$ and the line with equation $y = \frac{1}{2}$. The unit of length on each axis is one centimetre.

A uniform solid, S, is formed by rotating R through 180° about the y-axis.

Given that the volume of S is $\frac{15\pi}{4}$ cm^3 and using algebraic integration,

(a) show that the centre of mass of S is exactly 1.4 cm from the x-axis.

(5)

A uniform solid hemisphere of radius 2 cm, with the same density as S, is joined to S to form the composite solid B. The plane face of the hemisphere coincides with the larger plane face of S.

The plane face of B rests on a rough inclined plane that is at an angle α° to the horizontal. The inclined plane is sufficiently rough to prevent B from slipping.

Given that B is on the point of toppling,

(b) find the value of α.

(7)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 12 marks)

7.

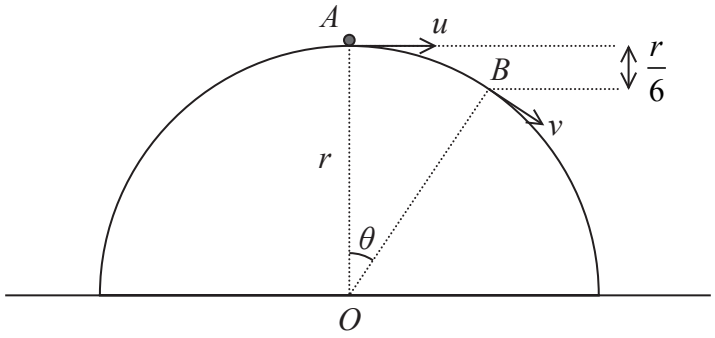

Figure 6

A smooth solid hemisphere, of radius r and centre O, is fixed with its plane face on a horizontal table. The highest point on the surface of the hemisphere is A.

A particle, P, of mass m is projected horizontally with speed u, where $u < \sqrt{gr}$, from A and moves along the surface of the hemisphere.

The particle leaves the surface of the hemisphere at the point B, which is a vertical distance $\frac{r}{6}$ below the level of A. The speed of P at the instant it reaches B is v, and the angle AOB is θ, as shown in Figure 6.

Assuming that air resistance may be ignored,

(a) show that $v^2 = \frac{5gr}{6}$

(4)

(b) explain the need for the restriction $u < \sqrt{gr}$

(1)

(c) find u in terms of g and r

(4)

(d) find the direction of motion of P at the instant it hits the table.

(5)

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

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